Bribery in Balanced Knockout Tournaments

Christine Konicki,
Virginia Vassilevska Williams

ABSTRACT

- Balanced knockout tournaments comprise a common format for sporting competitions, elections, and pairwise decision-making. We investigated the computational complexity of arranging the tournament's initial seeding and bribing players to lose, guaranteeing a favorite player's victory.
- **Question 1:** How do we bribe and seed a tournament described by a monotonic matrix (a case for which the hardness of seeding without bribery is unknown) without making the bribes obvious?
- **Question 2:** In a tournament generated by the Condorcet Random (CR) model, if we wanted to always have a handful of the best players "in our pocket," what is the lower bound on how many we would need so that we could efficiently find a winning seeding?

METHODS

1. Developed an NP-hardness reduction from an instance of the tournament fixing problem with bribery (BTFP) to an instance of Vertex-Cover
2. Proved a lower bound on how many players to bribe for a tournament with an easy-to-find winning seeding using Chernoff bound, structural conditions for quickly finding seedings, CR model.

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(1) Finding a winning seeding for a monotonic tournament is NP-complete if we can bribe players.

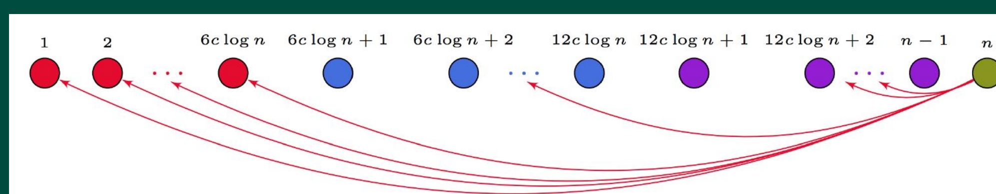
	v^*	v_j	e_j	$f_j^{r'}$	$h_{e_j}^{t'}$	$h_{f_j^{r'}}^{t'}$	$h_*^{t'}$
v^*	$-$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
v_i	ϵ	$1-\epsilon$ if $i<j$, ϵ o.w.	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
e_i	ϵ	ϵ	ϕ	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
f_i^r	ϵ	ϵ	ϵ	ϕ	ϕ	ϕ	$1-\epsilon$
$h_{e_i}^t$	ϵ	ϵ	ϵ	ϕ	ϕ	ϕ	$1-\epsilon$
$h_{f_i^r}^t$	ϵ	ϵ	ϵ	ϕ	ϕ	ϕ	$1-\epsilon$
h_*^t	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϕ

Matrix P pre-bribery. The bribes are indicated by the colored cells

	v^*	v_j	e_j	$f_j^{r'}$	$h_{e_j}^{t'}$	$h_{f_j^{r'}}^{t'}$	$h_*^{t'}$
v^*	$-$	1	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$	1
v_i	0	1 if $i<j$, 0 o.w.	1 if v_i covers e_j, $1-\epsilon$ o.w.	1	$1-\epsilon$	$1-\epsilon$	$1-\epsilon$
e_i	ϵ	0 if v_j covers e_i, ϵ o.w.	ϕ	1	1 if $i=j$, $1-\epsilon$ o.w.	$1-\epsilon$	$1-\epsilon$
f_i^r	ϵ	0	0	ϕ	ϕ	1 if $f_i^r = f_j^{r'}$, ϕ o.w.	$1-\epsilon$
$h_{e_i}^t$	ϵ	ϵ	0 if $i=j$, ϵ o.w.	ϕ	ϕ	ϕ	$1-\epsilon$
$h_{f_i^r}^t$	ϵ	ϵ	ϵ	0 if $f_i^r = f_j^{r'}$, ϕ o.w.	ϕ	ϕ	$1-\epsilon$
h_*^t	0	ϵ	ϵ	ϵ	ϵ	ϵ	ϕ

Matrix P' post-bribery. The bribes are indicated by the colored cells

(2) For almost all *n*-player tournaments generated by the CR model, if one bribes the best *6c log(n)* players to lose to any player v*, a winning seeding for v* can found in polynomial time.



- Stronger players tend to beat weaker players; monotonic tournament is more realistic.
- We bribe to yield an ε-monotonic matrix, nearly identical to the original; fixing this new tournament is known to be NP-complete.
- Benefits tournament organizer, as the bribes are difficult to detect.

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- When we bribe these top players, the tournament can be divided into sets **H**, **A**, **I**, and **{v*}** where:
  - **v*** is the winner and a king,
  - **A** contains all the players **v*** beats,
  - **H** contains all the players with ranks between 6c log(n) + 1 and 12c log(n) not in **A**, and
  - **I** contains all the other tournament players.
- This is the condition required for efficiently finding a winning seeding.